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R & D, HUMAN, AND PHYSICAL CAPITAL IN NEOCLASSICAL GROWTH

by Hans Brems

Abstract

At a natural rate of unemployment and a frozen capital stock early monetarist and New Classical supply-side equilibria found the natural supply of goods to be immune to monetary and fiscal policy.

The present paper considers three forms of capital stock.

First a human capital stock of accumulated flows of education.

Second a knowledge capital stock of accumulated flows of R & D.

Third a conventional capital stock of accumulated flows of physical investment. The paper unfreezes all three forms and finds physical output far from immune to monetary and fiscal policy.

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I. INTRODUCTION

The original Solow (1956) growth model knew only conventional physical capital stock. Technological progress was exogenous, falling as manna from heaven and blowing up the entire production function. The model was a standing invitation to endogenize technological progress. After all know-how can be created by R & D and disseminated by education.

Recent literature has accepted the invitation. Griliches (1973), (1979), (1988) saw a knowledge capital of accumulated R & D and estimated its productivity. The most recent estimate is Lichtenberg-Siegel (1991). Kendrick (1976) saw one-half of the 1969 U.S. capital stock as a human capital stock of accumulated education. The most recent estimate of its productivity is Mankiw-Romer-Weil (1990).

The present paper sees recent literature in a theoretical perspective. The original Solow model is extended to include three forms of capital stock. First a human capital stock of

accumulated flows of education. Second a knowledge capital stock of accumulated flows of R & D. Third a conventional capital stock of accumulated flows of physical investment. In the Solow tradition all three are considered immortal. Investment in human capital is public. Investment in knowledge and physical capital is private and optimized by maximizing present net worth, hence can be crowded out by an interest mechanism.

The model is simple enough to be solved for its levels and its growth rates. Solutions are found to be sensitive to monetary and fiscal policy.

II. THE MODEL

1. Variables

C ≡ consumption

D ≡ demand for money

E ≡ flow of education

G ≡ government purchase of goods

g ≡ proportionate rate of growth

H ≡ stock of human capital

I ≡ flow of physical investment

J ≡ flow of R & D investment

K ≡ stock of knowledge capital

k ≡ present gross worth of another unit of knowledge capital

κ ≡ marginal productivity of knowledge capital stock

L ≡ labor employed

P ≡ price of good

R ≡ tax revenue

r ≡ before-tax nominal rate of interest

ρ ≡ aftertax real rate of interest

S ≡ stock of physical capital

s ≡ present gross worth of another unit of physical capital

 $\sigma \equiv \text{marginal productivity of physical capital stock}$

v ≡ money salary rate

w ≡ money wage rate

X ≡ physical output

Y = money national income

y ≡ money disposable income

2. Parameters

a ≡ joint factor productivity

 α , β , γ , δ = exponents of a Cobb-Douglas production function

b ≡ a demand parameter

c ≡ propensity to consume

 $\eta \equiv reciprocal$ of Marshallian price elasticity of demand

F ≡ available labor force

f ≡ fraction of government purchase allocated to education

 $\lambda \equiv$ "natural" employment rate

M ≡ supply of money

m ≡ reciprocal of the velocity of money

T = tax rate

All parameters are stationary except a, F, and M, whose growth rates are stationary.

3. Definitions

Define the proportionate rate of growth of variable v as

$$g_v \equiv \frac{dlog_e v}{dt} \tag{1}$$

Human capital stock H is accumulated investment E in education. Knowledge capital stock K is accumulated investment J in R & D. Conventional capital stock S is accumulated physical investment I. Consequently under immortal capital stocks:

$$E \equiv g_{H}H \tag{2}$$

$$J \equiv g_{K}K \tag{3}$$

$$I \equiv g_S S \tag{4}$$

4. Product

Like a Solow economy our economy produces a single good but makes alternative uses of it. Regardless of its use let the good be produced in the same way under a Cobb-Douglas production function common to all firms

$$X = \alpha L^{\alpha} H^{\beta} K^{\gamma} S^{\delta}$$
 (5)

where α , β , γ , and δ are positive proper fractions summing to one and where a is joint factor productivity growing at the rate g_a .

Such a linearly homogeneous production function rules out nonpure competition rooted in economies of scale but not nonpure competition rooted in product differentiation. R & D creates productivity advantages to be enjoyed within niches sheltered from rivals. Within such niches let all firms imagine themselves facing the same constant-elasticity demand function

$$P = bX^{\eta}$$

where $0 \ge \eta > -1$ is the reciprocal of a Marshallian price elasticity of demand. Our solutions will display the factor $0 < 1 + \eta \le 1$ to be thought of as a measure of the degree of competition in the goods market. Under pure competition firms cannot control price, and the equality sign holds: $1 + \eta = 1$. The less pure competition in the goods market is, the lower the factor $1 + \eta$.

5. Demand for Labor

Let labor be hired at the money wage rate w. A firm will maximize its aftertax profits by equating the money wage rate with the marginal-revenue productivity of labor hired:

$$w = (1 + \eta) P \frac{\partial X}{\partial L}$$

Differentiate, rearrange, and write firm demand for labor as a function of the real wage rate:

$$L = \alpha (1 + \eta) X / (w/P)$$
 (6)

As Lindbeck-Snower (1989: 373) observed, firms facing the same production function, demand function, and factor prices will behave alike, i.e., decide on the same factor use and physical output. In that case the demand for labor (6) is easily aggregated.

6. Supply of Labor. The "Natural" Real Wage Rate

Facing such aggregate demand for labor (6), how much do unions choose to supply? Friedman's answer (1968) was his "natural" rate of unemployment to which current labor-market literature adds institutional color: Lindbeck and Snower (1986) and Blanchard and Summers (1988) distinguish between "insiders," who are employed hence decision-making, and "outsiders," who are unemployed hence disenfranchised. Let insiders accept the "natural" employment rate λ where $0 < \lambda \le 1$. We may think of the rate λ as a measure of the degree of competition in the labor market: under pure competition unions cannot control real wage

rates, and the equality sign holds, λ = 1. The less pure competition in the labor market is, the lower the rate λ .

The rate λ is "natural" in the sense that if L > λF insiders will insist on a higher real wage rate. If

$$L = \lambda F \tag{7}$$

they will be happy with the existing one. If L < λF they will settle for a lower one.

Given their natural rate of employment, insiders will be happy with a "natural" real wage rate found by inserting (7) into (6):

$$w/P = \alpha (1 + \eta) X/(\lambda F)$$
 (8)

At frozen capital stocks H, K, and S, X would decline in less than proportion to λ , and labor can have a higher natural real wage rate by accepting a lower λ .

7. Demand for Services of Human Capital

Let services of human capital be hired at the money salary rate v. A firm will maximize its aftertax profits by equating the money salary rate with the marginal-revenue productivity of services hired:

$$V = (1 + \eta) P \frac{\partial X}{\partial H}$$

Differentiate, rearrange, and write firm demand for services of human capital as a function of the real salary rate:

$$H = \beta (1 + \eta) X / (v/P)$$
 (9)

Again firms facing the same production function, demand function, and factor prices will decide on the same factor use and physical output, and demand for services of human capital (9) is easily aggregated.

8. Supply of Services of Human Capital. Real Salary Rate

Human capital stock H is accumulated flows of investment E in education, and the flow of education is decided upon by the government. Let the market for services of human capital be clearing at whatever human capital has accumulated—in sec. 10 below we shall see how much. Find the resulting real salary rate by rearranging (9):

$$V/P = \beta (1 + \eta) X/H \tag{10}$$

9. Knowledge and Physical Capital Stocks: Private Optimization

Knowledge and physical capital stocks K and S are privately optimized. We begin with their marginal productivities

$$\kappa \equiv \frac{\partial X}{\partial K} = \gamma \frac{X}{K} \tag{11}$$

$$\sigma \equiv \frac{\partial X}{\partial S} = \delta \frac{X}{S} \tag{12}$$

Their marginal-revenue productivities will then be $(1 + \eta)\kappa P$ and $(1 + \eta)\sigma P$, respectively. Such marginal-revenue productivities of immortal capital stocks will be marginal net returns taxed at rate T. Let nominal interest expense be taxdeductible, then money may be borrowed at an aftertax nominal rate of interest (1 - T)r. Discount future cash flows at that rate. Define present gross worths k and s of another unit of knowledge or physical capital stock, respectively, as the present worth at time τ of all its future aftertax marginal-revenue productivities

$$k(\tau) \equiv \int_{\tau}^{\infty} (1 + \eta)(1 - T)\kappa(t)P(t)e^{-(1 - T)r(t - \tau)}dt$$

$$s(\tau) \equiv \int_{\tau}^{\infty} (1 + \eta)(1 - T)\sigma(t)P(t)e^{-(1 - T)r(t - \tau)}dt$$

In (38) we shall see that $g_K = g_S = g_X$ and in (40) that price is growing at the stationary rate g_B . Consequently

$$\kappa(t) = \kappa(\tau)$$

$$\sigma(t) = \sigma(\tau)$$

$$P(t) = P(\tau)e^{g_{p}(t - \tau)}$$

Define the aftertax real rate of interest

$$\rho \equiv (1 - T) r - g_{\scriptscriptstyle P} \tag{13}$$

Insert all this into $k(\tau)$ and $s(\tau)$, move $(1+\eta)$, (1-T), $\kappa(\tau)$, $\sigma(\tau)$, and $P(\tau)$ outside the integral signs, assume $\rho>0$, and take the integrals

$$k = (1 + \eta) (1 - T) \kappa P / \rho$$

$$s = (1 + \eta) (1 - T) \sigma P / \rho$$

Present net worth of another unit of capital stock is its present gross worth <u>minus</u> its price. In our one-good economy that price is P, so

$$k - P = [(1 + \eta)(1 - T)\kappa/\rho - 1]P$$

$$s - P = [(1 + \eta)(1 - T)\sigma/\rho - 1]P$$

Optimized capital stock is the size of stock at which the present net worth of another unit is zero:

$$(1 + \eta) (1 - T) \kappa = \rho \tag{14}$$

$$(1 + \eta) (1 - T) \sigma = \rho \tag{15}$$

from which we see that $\kappa = \sigma$: at their optimized sizes knowledge and physical capital stocks have equal marginal productivities. To find those sizes insert (11) and (12) into (14) and (15), respectively:

$$K = \gamma (1 + \eta) (1 - T) X/\rho \tag{16}$$

$$S = \delta (1 + \eta) (1 - T) X/\rho$$
 (17)

Again factor uses (16) and (17) are easily aggregated. Divide such aggregated (16) by (17) and see that $K/S = \gamma/\delta$ hence

$$g_{\kappa} = g_{S} \tag{18}$$

We turn to human capital stock, decided by government.

10. Human Capital Stock: Public Finance

With immortal capital stocks national income is the market value of physical output

$$Y \equiv PX \tag{19}$$

Such national income is taxed at rate T, so tax revenue is

$$R = TY (20)$$

where 0 < T < 1. As a first approximation let government finance a deficit by increasing the money supply. The government budget constraint then collapses into

$$GP - R = g_{M}M \tag{21}$$

As a first approximation [Friedman (1959)] let the demand for money be a function of money national income but not of the rate of interest:

$$D = mY (22)$$

where m > 0. Let the money market clear:

$$M = D \tag{23}$$

Into (21) insert (19), (20), (22), and (23) and find what purchase of goods the government can afford:

$$G = (T + g_{M}m) X \tag{24}$$

Let the government allocate the fraction f of it to education:

$$E = fG \tag{25}$$

Into (2) insert (24) and (25) and find human capital stock

$$H = f(T + g_M m) X/g_H$$
 (26)

11. Consumption

Define disposable income as national income <u>minus</u> tax revenue:

$$y \equiv Y - R \tag{27}$$

and let consumption be the fraction c of disposable real income:

$$C = cy/P \tag{28}$$

where 0 < c < 1. Into (28) insert (19), (20), and (27):

$$C = c(1 - T)X \tag{29}$$

12. Goods-Market Clearance

The single good of our one-good economy was consumed, invested in physical or knowledge capital, or purchased by government. Let the goods market clear:

$$X = C + I + J + G \tag{30}$$

We may now solve our system.

III. SOLUTIONS

1. Levels

The goods market is cleared by the aftertax real rate of interest. Into (30) insert (3), (4), (16), (17), (18), (24), and (29) and solve for the aftertax real rate of interest:

$$\rho = (\gamma + \delta) (1 + \eta) (1 - T) / A, \text{ where}$$

$$A \equiv \frac{(1 - c)(1 - T) - g_{M}m}{g_{S}}$$
 (31)

Into (16) and (17) insert our solution (31). Into (5) insert the result and (7) and (26) and solve for physical output

$$X = \alpha^{1/\alpha} \lambda F(\frac{\gamma}{\gamma + \delta})^{\gamma/\alpha} (\frac{\delta}{\gamma + \delta})^{\delta/\alpha} A(\gamma + \delta)/\alpha_B \beta/\alpha, \text{ where}$$

$$B \equiv f(T + g_M m)/g_H \tag{32}$$

Once we have solved for X the rest is easy. Solve for the sum of knowledge and physical capital by inserting (31) into the sum of (16) and (17):

$$K + S = AX \tag{33}$$

Solve for human capital by writing (26) as

$$H = BX \tag{34}$$

The real wage rate was

$$w/P = \alpha (1 + \eta) X/(\lambda F)$$
 (8)

Solve for price by inserting (19) into (22) and (23):

$$P = M/(mX) \tag{35}$$

Solve for the real salary rate by inserting (26) into (10):

$$v/P = \beta (1 + \eta)/B \tag{36}$$

2. Growth Rates

All parameters were said to be stationary except a, F, and M. Consequently the growth rates of our levels (8) and (31) through (36) will be

$$g_{p} = 0 \tag{37}$$

$$g_H = g_K = g_S = g_X = g_a/\alpha + g_P$$
 (38)

$$g_{w/P} = g_a/\alpha \tag{39}$$

$$g_P = g_M - (g_a/\alpha + g_F) \tag{40}$$

$$g_{v/P} = 0 (41)$$

3. Growth Accounting

Let an original two-factor Solow model have factor exponents 0.8 for labor and 0.2 for capital. Let its joint factor productivity, its labor, and its capital be growing at 0.016, 0.01, and 0.03, respectively. Then its output would be growing at 0.03, of which the exogenous growth rate 0.016 of joint factor productivity, left unexplained, would be 53 percent.

Adding endogenous human and knowledge capital we may sharply reduce that unexplained part. Under the not implausible values of table I physical output would be growing at the rate $g_{\chi}=g_a+\alpha g_F+\beta g_H+\gamma g_K+\delta g_S=0.03$ as before. But of $g_{\chi}=0.03$,

TABLE I. NOT IMPLAUSIBLE VALUES

<u>Parameters</u>

 $\alpha = 0.4$

 $\beta = 0.4$

 $\gamma = 0.1$

 $\delta = 0.1$

1 - c = 0.08

f = 0.25

 $g_a = 0.008$

 $g_F = 0.01$

 $g_{M} = 0.06$

m = 0.2

T = 0.25

<u>Variables</u>

$$g_{H} = g_{K} = g_{S} = g_{X} = 0.03$$

$$g_{w/P} = 0.02$$

 $g_a = 0.008$ would be only 27 percent. The unexplained part has been cut in half!

IV. PROPERTIES OF SOLUTIONS

1. Will Levels be Positive and Finite?

All levels (8) and (31) through (35) contain A and will be positive and finite as long as A > 0. Tobin's (1986) "debacle" will result if A = 0, i.e., if the deficit share g_M^m of national income swallows up the savings share (1-c)(1-T). In that case physical output, all capital stocks, and the real wage rate will turn zero. The aftertax real rate of interest and price will turn undefined and have the limits

$$\lim_{A \to 0} = \lim_{A \to 0} = \infty$$

2. A Crowding-Out Mechanism

Capital coefficients are the clue to crowding-out and are apparent in (33) and (34). The "private" capital coefficient is A, and A is always down when g_M or T is up: $\partial A/\partial g_M = -m/g_S < 0$; $\partial A/\partial T = -(1-c)/g_S < 0$. The "public" capital coefficient is B, and B is always up when g_M or T is up: $\partial B/\partial g_M = fm/g_H > 0$; $\partial B/\partial T = f/g_H > 0$. Per unit of output, then, public capital is crowding out private capital.

The crowding-out mechanism is now easy to see: the cost of capital is the aftertax real rate of interest (31), always up when g_{M} or T is up.

3. Supply-Side Equilibrium

Early supply-side equilibria by Friedman (1968), Lucas (1972), Sargent (1973), and Sargent-Wallace (1975) were static equilibria: capital stock remained frozen. At such frozen capital stock and a "natural" rate of unemployment physical output would be immune to monetary and fiscal policy.

We have unfrozen all our three capital stocks and made them endogenous. We find no immunity to monetary and fiscal policy: physical output (32) may go either way when g_M or T is up. Physical output depends both on A, which is down, and on B, which is up. And A comes with the exponent $(\gamma + \delta)/\alpha$ and B with the exponent β/α . Estimates of such exponents offered in recent literature by Griliches, Lichtenberg-Siegel, Mankiw-Romer-Weil, and others are crucial.

Which way physical output (32) will go is described by the signs of its two elasticities

$$\frac{\partial X}{\partial g_{M}} \frac{g_{M}}{X} = \left(\frac{\beta}{\alpha} \frac{f}{B} - \frac{\gamma + \delta}{\alpha} \frac{1}{A}\right) \frac{g_{M}m}{g_{S}} \tag{42}$$

$$\frac{\partial X}{\partial T}\frac{T}{X} = \left(\frac{\beta}{\alpha}\frac{f}{B} - \frac{\gamma + \delta}{\alpha}\frac{1 - c}{A}\right)\frac{T}{g_s} \tag{43}$$

At the not implausible parameter values of table I the elasticity (42) will be -0.08 and the elasticity (43) 0.75. Why

should the effect of monetary policy be negative but the effect of fiscal policy positive?

First compare the effects of monetary and fiscal policy upon A. At the parameter values of table I the elasticity of A is -0.25 with respect to g_M and -0.42 with respect to T. In other words a one percent higher tax rate T will have a 1.7 times more powerful (negative) effect upon A than would a one percent higher g_M .

Second compare the effects of monetary and fiscal policy upon B. At the parameter values of table I the elasticity of B is 0.046 with respect to g_M and 0.95 with respect to T. In other words a one percent higher tax rate T will have a 21 times more powerful (positive) effect upon B than would a one percent higher g_M .

In short, the chance that its positive effect upon the "public" capital coefficient B will swamp its negative effect upon the "private" capital coefficient A is better for fiscal than for monetary policy.

Two essential propositions of a supply-side equilibrium did survive. The first was the immunity of employment (7) to public

policy. The second was the working of an interest and price mechanism. As we saw, the labor market may not clear: there was a natural rate of employment $0 < \lambda \le 1$. But two equilibrating variables, i.e., the aftertax real rate of interest ρ and price P did clear the goods and money markets (30) and (23), respectively.

One last thing. Our model assumed competition to be nonpure. Was nonpure competition of any consequence?

4. Nonpure Competition in the Goods Market

Our factor prices (8), (31), and (36) are in direct proportion to 1 + η hence are the lower the less pure competition is in the goods market. No other level has any 1 + η in it.

Less pure competition, then, merely depresses factor prices but has no effect upon factor use (7), (33), or (34) or upon physical output (32). Less pure competition simply redistributed income in favor of entrepreneurs at the expense of wage and salary earners and capitalists. Such redistribution may be seen as the result of successful R & D.

5. Nonpure Competition in the Labor Market

Factor use (7), (33), and (34) and physical output (32) are in direct proportion to λ hence are the lower the less pure competition is in the labor market. Price (35) is in inverse proportion to physical output X, hence to λ and is the higher the less pure competition is in the labor market. No other level has any λ in it.

Such invariance of factor prices (8), (31), and (36) with λ sheds new light on union policy. As we observed in (8), at frozen capital stocks H, K, and S labor could have a higher real wage rate by accepting a lower natural rate λ of employment. In the long run, i.e., at unfrozen capital stocks H, K, and S, X is in direct proportion to λ , the natural rate λ will cancel in (8), and labor can have a no higher natural real wage rate by accepting a lower λ . The economy is simply reduced to a lower scale: accumulating proportionately less capital stock and producing proportionately less output. The economy is impoverishing itself. Labor doesn't benefit, nobody benefits.

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